Propagation Characteristics of TE-Waves Guided by Thin Films Bounded by Nonlinear Media

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***Abstract*—The behavior of nonlinear TE-waves guided by asymmetrical dielectric slab waveguides, having a generalized nonlinear substrate with a permittivity of the form $\epsilon \sim |\vec{E}|^{\delta}$, is analyzed. Using an analytical solution for the electromagnetic fields in this nonlinear media, the dispersion relations, electric field profiles, and cutoff frequencies are illustrated and discussed.**

I. Introduction

SEMICONDUCTOR multilayer systems consisting of a sequence of GaAs–AlAs films exhibit very strong optical nonlinearities [1]. These strong nonlinearities are an essential prerequisite for the formation of nonlinear guidedwaves (NGW's) along the interfaces of layered structures. During the past years, considerable literature has emerged, reporting on this particular type of guided wave, which evokes field-induced changes in the refractive index due to the optical effect that can match the steps in the linear refractive index across the interfaces. Interesting peculiarities of NGW's, as power-dependent propagation constants and field profiles, offer some potential for all-optical signal processing (see, e.g., [1]–[11] and references therein). The authors of [7] indicate that strongly nonlinear waves could also be realized by using properly designed waveguiding structures.

The principal effort was initially concentrated upon the investigation of TE-waves propagating along single-film structures bounded by semiinfinite Kerr-like nonlinear media. The formulation of this problem results in a propagation constant being treated as an implicit function of the modulus of a Jacobian elliptic function (e.g., [5]) or by numerical methods (e.g., [1]–[4]).

However, in real media, many materials exhibit a refractive index which varies with the optical field raised to a power other than two, e.g., semiconductor structures [1], [10]–[12]. Detailed knowledge of the propagation characteristics of guided waves supported by arbitrarily nonlinear waveguides is crucial for a better optical device design.

Very recently, we have developed an analytical solution for the electromagnetic fields in generalized nonlinear media [9] that has offered the opportunity to study the dispersion relations and the field properties of NGW's in media with generalized nonlinearities, which describes field solutions and dispersion relations in closed form. This paper presents the behavior of TE-waves guided in a structure which is composed of three layers, consisting of the nonlinear substrate with a permittivity of the form $\epsilon \sim |\vec{E}|^{\delta}$, a linear film, and a linear cladding. The explicit analytical expressions for the propagation characteristics, the power-dependent dispersion, and the propagation conditions of TE-waves are given. In the discussion, losses are omitted in order to obtain universal results.

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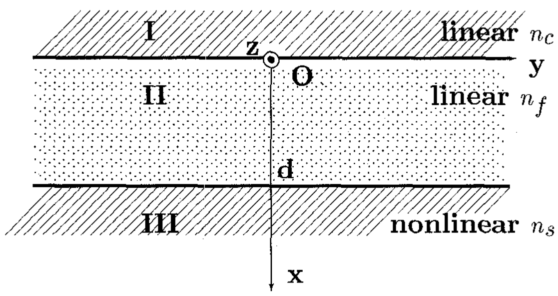

Fig. 1. Waveguide structure.

II. Electromagnetic Fields in the Media

Fig. 1 shows the considered three-layer structure which consists of a thin optically linear film of thickness d and refractive index n_f, a linear cover with refractive index n_c, and a semiinfinite nonlinear medium forming the substrate which has a dielectric profile

$$\epsilon = \epsilon_o[n_s^2 + \alpha|\vec{E}|^{\delta}] \tag{1}$$

with ϵ_o being the free-space permittivity, n_s the refractive index of the material for $|\vec{E}| = 0$, and α an arbitrary constant. Only this special form of a field-dependent permittivity, which is dependent on the magnitude of the electric strength, is considered here. Here, δ can be an arbitrary real number. Assuming the guide is infinite in the y–direction, then the resulting fields will be independent of y. For TE-waves, $E_z = 0$ and only $E_y \neq 0$.

The z–direction is assumed to be the propagating direction, and possible waves in this direction are described by the complex function $e^{-j\beta z}$. We will focus our attention on wave solutions of the form $\vec{E} \sim e^{j(\omega t - \beta z)}$.

For the three regions shown in Fig. 1, the wave equations are

$$\frac{d^2E_y}{dx^2} - (\beta^2 - k_o^2 n_c^2)E_y = 0, \qquad x \leq 0 \tag{2}$$

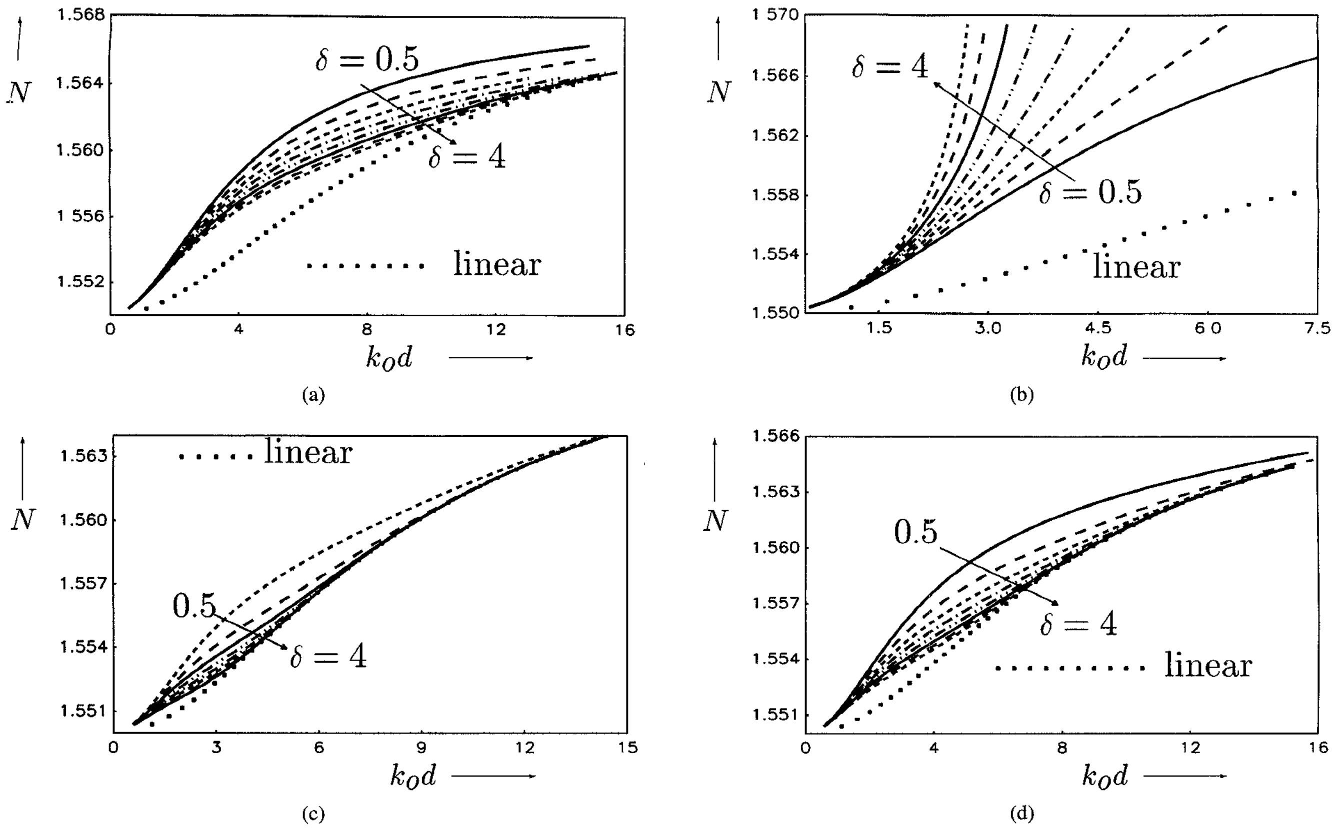

Fig. 2. The dispersion relations of the first guided mode TE_o with various values of δ and a step width of $\Delta\delta = 0.5$, and $n_s = n_c = 1.55$, $n_f = 1.57$. (a) $x_o = 0.7d$, (b) $x_o = 1.5d$, (c) $x_o = -1.5d$, (d) $x_o = 0$.

$$\frac{d^2E_y}{dx^2} + (k_o^2 n_f^2 - \beta^2)E_y = 0, \qquad 0 \le x \le d \tag{3}$$

$$\frac{d^2E_y}{dx^2} - (\beta^2 - k_o^2 n_s^2 - k_o^2 \alpha |E_y|^\delta)E_y = 0, \qquad d \le x. \tag{4}$$

In this paper, we discuss only the guided waves. Using the solution of (4) given in [9], the electric fields in the three regions are

$$E_y = A\cosh^{-2/\delta}\left[\frac{\delta}{2}k_3(x - x_o)\right], \qquad x \ge d$$

$$E_y = A\cosh^{-2/\delta}\left[\frac{\delta}{2}k_3(d - x_o)\right] \cdot \frac{\cos(k_2 x) + \frac{k_1}{k_2}\sin(k_2 x)}{\cos(k_2 d) + \frac{k_1}{k_2}\sin(k_2 d)}, \qquad 0 \le x \le d$$

$$E_y = A\cosh^{-2/\delta}\left[\frac{\delta}{2}k_3(d - x_o)\right] \cdot \frac{e^{k_1 x}}{\cos(k_2 d) + \frac{k_1}{k_2}\sin(k_2 d)}, \qquad x \le 0 \tag{5}$$

and the dispersion relation is

$$k_2 d = m\pi + \tan^{-1}\frac{k_1}{k_2} + \tan^{-1}\frac{k_3'}{k_2} \tag{6}$$

where

$$A = E_o\left(\frac{1}{\alpha|E_o|^\delta}\,\frac{2+\delta}{2}\,\frac{k_3^2}{k_o^2}\right)^{1/\delta} \tag{7}$$

$$k_o^2 = \omega^2\epsilon_o\mu_o, \qquad k_1^2 = \beta^2 - k_o^2 n_c^2,$$
$$k_2^2 = k_o^2 n_f^2 - \beta^2, \tag{8}$$

$$k_e^2 = \beta^2 - k_o^2 n_s^2, \qquad k_3' = k_3 \tanh\left[\frac{\delta}{2}k_3(d - x_o)\right]. \tag{9}$$

x_o is a parameter given by the initial exciting conditions, and E_0 is a constant magnitude of the electric field.

If the substrate in region III is linear ($\alpha = 0$), the dispersion relation is [7]

$$k_2 d = m\pi + \tan^{-1}\frac{k_1}{k_2} + \tan^{-1}\frac{k_3}{k_2} \tag{10}$$

with all parameters defined in (8) and (9). That is, if region III is a nonlinear medium, the dispersion relation of the guided TE-waves has the same form as that of the linear case, except that k_3' is used instead of k_3.

From (5) it is known that the magnitude of the electric field of guided waves is dependent on the parameters $N = \beta/k_o$, n_s, α, and δ. If we let the magnitude of the electric field at $x = d$ be E_d, from (5) and (7) it follows that

$$\alpha|E_d|^\delta = \frac{2+\delta}{2}\frac{k_3^2}{k_o^2}\Big/\cosh^2\left[\frac{\delta}{2}k_3(d - x_o)\right]. \tag{11}$$

Now, only the modes whose electric fields at $x = d$ have the magnitude determined in (11) can exist in this structure. In other words, the propagation parameter $N = \beta/k_o$ is not only dependent on the dielectric parameters n_s, α, δ, and the angular frequency ω, but it is also a function of the magnitude

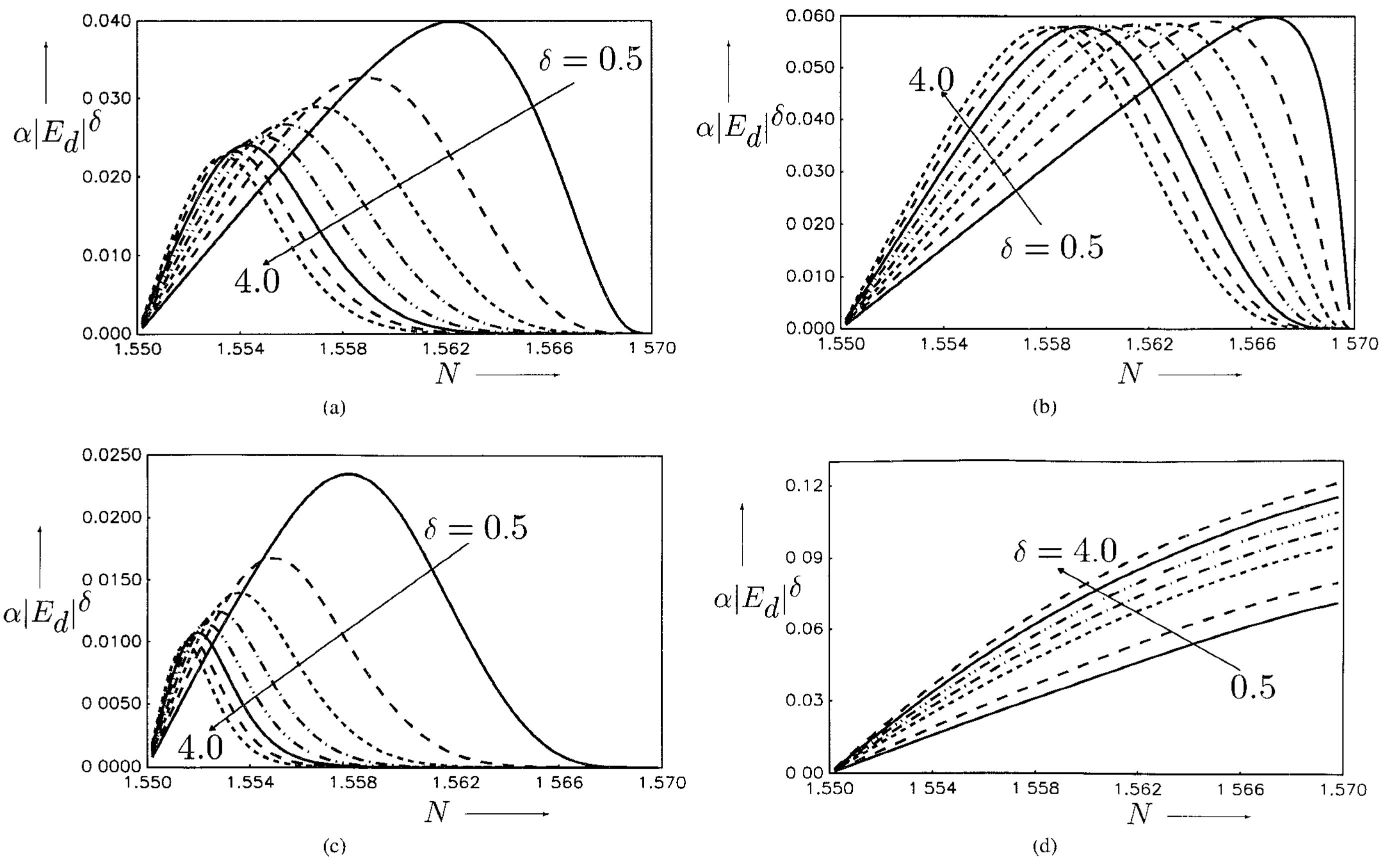

Fig. 3. The dependences of N versus $\alpha|E_d|^\delta$ with various values of δ for $n_s = n_c = 1.55$, $n_f = 1.57$. Here the step of $\Delta\delta$ is 0.5. (a) $x_o = 0$, (b) $x_o = 0.7d$, (c) $x_o = -1.5d$, (d) $x_o = 1.5d$.

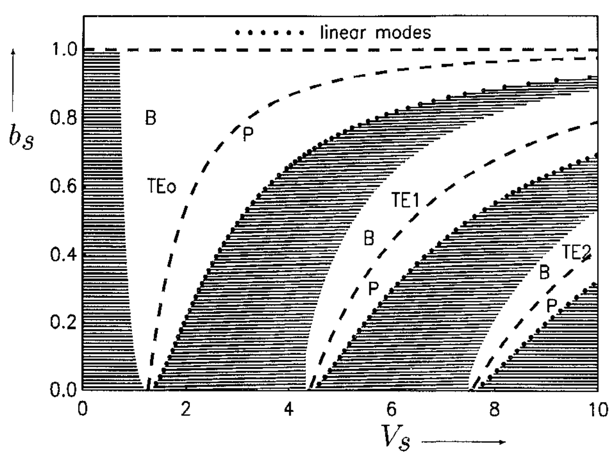

Fig. 4. Allowed and forbidden bands for TE guided propagation in the structure with the generalized nonlinear substrate. P: pure guided waves (PWB); B: bulged guided waves (BWG); shady region: forbidden region; and dashed lines: $x_o = d$. Here $a_s = 10$. Critical values of V_s are $m\pi+\tan^{-1}\sqrt{a_s}$, $n_s > n_c$.

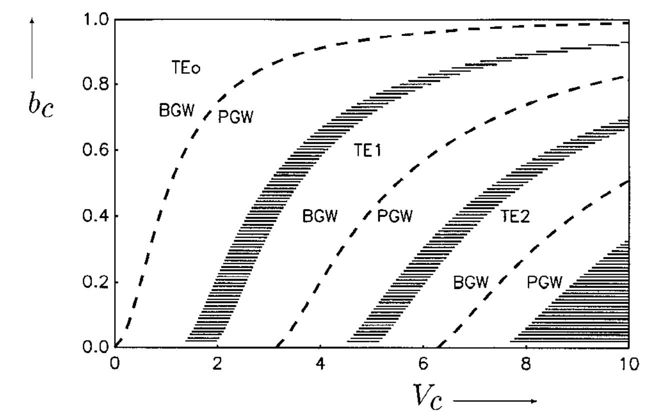

Fig. 5. Allowed and forbidden bands for TE guided propagation in the structure with the generalized nonlinear substrate. Shady region: forbidden region; dashed lines: dispersion curves for $x_o = d$. Here, $a_c = 10$ and $n_c > n_s$.

of the electric field. x_o is a factor related to the initial exciting conditions. If E_d and the nonlinear parameters α, δ are given, x_o and β/k_o can be determined. Under these conditions, only this mode exists in the guide. If $x_o > d$, there is a maximum of the electric field in the nonlinear substrate region III at $x = x_o$. In the region $x > x_o$, the field decreases exponentially; but in the region $d < x < x_o$, the field increases, and the point of the field maximum is invariable. If $\delta = 2$, the Kerr-like nonlinear substrate, the field distributions, and the dispersion relations from (5) and (6) are identical to those in [7]. The modes propagating in this structure with generalized nonlinear substrate can be divided into the following three groups, as in the cases of Kerr-like nonlinear substrate [7].

1) *Pure Guided Waves (PGW):* Now $x_o < d, N < n_f$, and there is no field maximum in the region III.

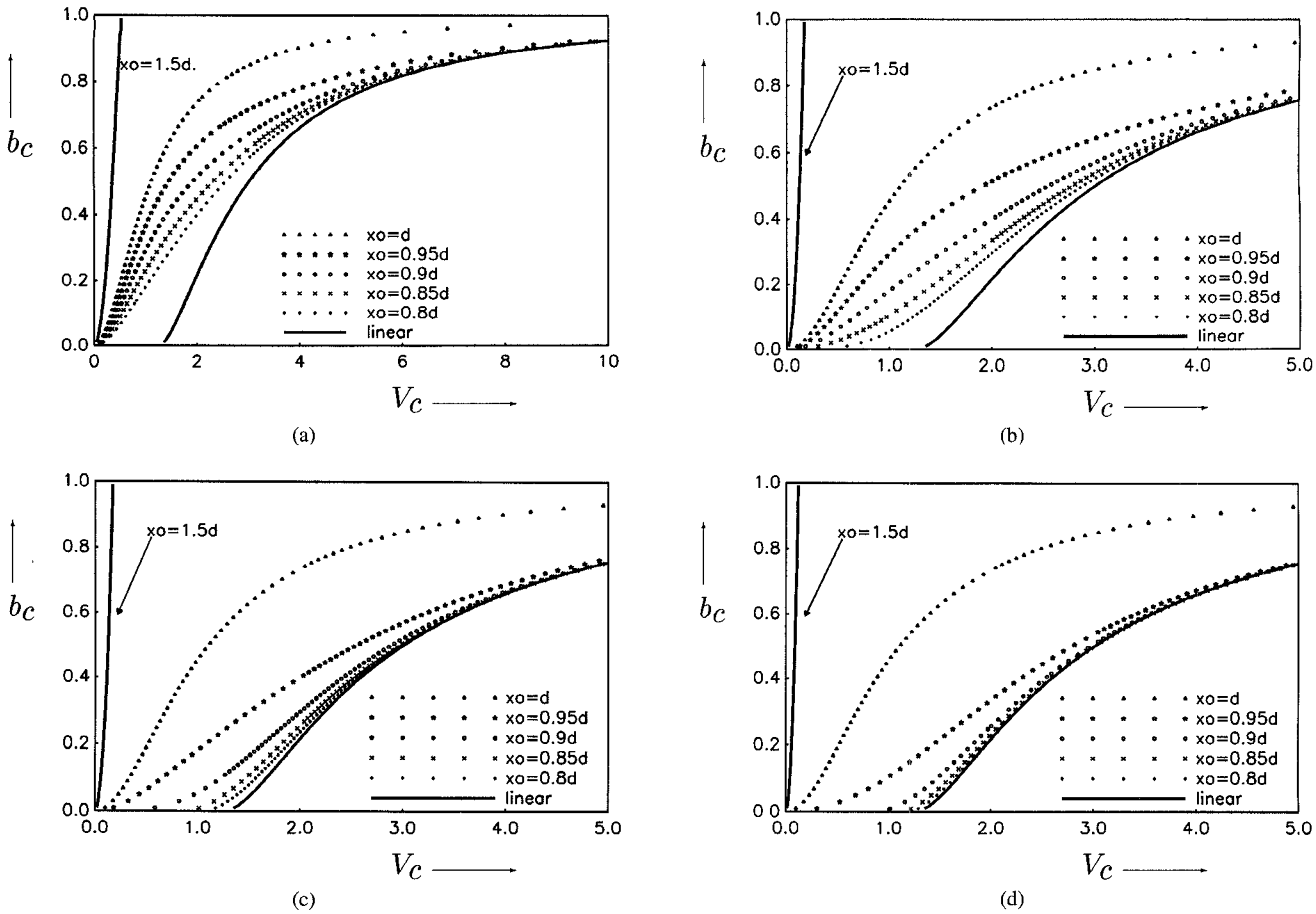

Fig. 6. The curves of $b_c \sim V_c$ with various values of δ and x_o for $n_c > n_s$. Here, $a_c = 9$. (a) $\delta = 0.5$, (b) $\delta = 1$, (c) $\delta = 2$, (d) $\delta = 3$.

2) *Bulged Guided Waves (BGW):* Now $x_o > d, N < n_f$, and there is a field maximum in the region III at $x = x_o$.
3) *Surface Waves (SW):* Now $N > n_f$.

III. Results and Discussion

From (6) and (10), the parameters k_2^{NL}, k_2^L can be determined for the nonlinear structure (NL) and the linear structure (L), respectively, and

$$k_2^L d - k_2^{NL} d = \tan^{-1}\frac{k_3}{k_2} - \tan^{-1}\frac{k_3'}{k_2} > 0 \qquad (12)$$

because of $k_3 > k_3'$ for any fixed N, n_c, n_f, n_s. That is, if the parameter N and the dielectric properties are the same in both the linear and the nonlinear structures, the angular frequency ω in the nonlinear case is lower than that in the linear case with the same structure for the guided waves.

If $x_o = d$, we have $k_3' = 0$. From (6)

$$k_2 d = m\pi + \tan^{-1}\frac{k_1}{k_2} = m\pi + \tan^{-1}\sqrt{\frac{N^2 - n_c^2}{n_f^2 - N^2}}. \qquad (13)$$

Now, the dispersion relation is independent of the parameter n_s.

Let $n_s = n_c$ be a "symmetrical" guide, and $n_s \neq n_c$ be an "asymmetrical" guide, then the following results can be discussed.

1) The "Symmetrical" Guide: Now, the cutoff wavenumbers are identical to those in the linear cases. In Fig. 2, the dispersion relations of the first guided mode TE_0 are shown for a given value of x_o, and $\delta = 0.5$, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0, respectively. The results show that all N-values of the nonlinear cases are above the linear case. For the case $x_0 > d$, there are greater differences between the linear and the nonlinear cases. For greater δ and $k_o d$, the second mode TE_1 will become the fundamental mode, unlike in the linear cases in which the mode TE_0 is always the fundamental mode in the linear structures.

Fig. 3 shows the dependences of αE_d^δ versus N for various values of δ, for an x_o, and $\delta = 0.5$, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, The results show that for a given magnitude of the electric field E_d at $x = d$ and the parameters of nonlinearity, there are two modes having TE_0 properties with different propagation factors N and exciting frequency in the structure. Fig. 3 shows that if δ and x_o are fixed, there is a maximum value of $\alpha|E|_d^\delta$. Only if $\alpha|E|_d^\delta$ is less than the maximum value are there related N. Corresponding the maximum value, there is only one existed mode. The position of the maximum value of $\alpha|E|_d^\delta$ is dependent on the factors δ and x_o.

2) The "Asymmetrical" Guide: In terms of the well-known normalized parameters [7]: the normalized thickness (V), symmetry measure (a), and the normalized effective index

(b) of the waveguide

$$V_s = k_o d\sqrt{n_f^2 - n_s^2}, \qquad a_s = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2},$$

$$b_s = \frac{N^2 - n_s^2}{n_f^2 - n_s^2} \tag{14}$$

the effective dispersion relation is

$$V_s\sqrt{1-b_s} = m\pi + \tan^{-1}\sqrt{\frac{b_s + a_s}{1-b_s}} + \tan^{-1}\left(\sqrt{\frac{b_s}{1-b_s}} \cdot \tanh\left[\frac{\delta}{2}V_s\sqrt{b_s}\left(1 - \frac{x_o}{d}\right)\right]\right) \tag{15}$$

for $n_s > n_c$ or

$$V_c = k_o d\sqrt{n_f^2 - n_c^2}, \qquad a_c = \frac{n_c^2 - n_s^2}{n_f^2 - n_c^2},$$

$$b_c = \frac{N^2 - n_c^2}{n_f^2 - n_c^2} \tag{16}$$

and

$$V_c\sqrt{1-b_c} = m\pi + \tan^{-1}\sqrt{\frac{b_c}{1-b_c}} + \tan^{-1}\left(\sqrt{\frac{b_c + a_c}{1-b_c}}\tanh\left[\frac{\delta}{2}V_c\sqrt{b_c + a_c} \cdot \left(1 - \frac{x_o}{d}\right)\right]\right) \tag{17}$$

for $n_s < n_c$.

For $n_s > n_c$, the cutoff wavenumbers are the same as those of the linear cases

$$k_c d = (m\pi + \tan^{-1}\sqrt{a_s})/\sqrt{n_f^2 - n_s^2}. \tag{18}$$

In Fig. 4, we have plotted the allowed and forbidden regions in the $V_s \sim b_s$ plane for guided propagation with all possible parameters x_o and δ to occur; here, $a_s = 10$ and $n_s > n_c$. For $\delta = 2$ (Kerr-like media), it is the same as the Fig. 3 given by Torres *et al.* in [7], who have studied the structure with Kerr-like nonlinearity. The results show that if δ or/and x_o is large, the behavior of the BGW is very different from that of PGW's. For the linear modes or nonlinear PGW's, only the modes whose wavenumbers are greater than their cutoff wavenumbers can exist in the guides; but now for the first bulged guided waves TE_0 with large δ or/and x_o, only the modes whose wavenumbers are less than their cutoff wavenumbers can exist in the guides. All dispersion curves with various δ and x_o are located in the allowed regions shown in Fig. 4. If a_s is varied, we can also get the allowed and forbidden bands similar to Fig. 4.

For $n_c > n_s$, from (17), it is known that the cutoff wavenumbers of the nonlinear cases are different from those of the linear case, and they are a function of the parameter δ and x_o

$$V_c = m\pi + \tan^{-1}\left(\sqrt{a_c}\tanh\left[\frac{\delta}{2}V_c\sqrt{a_c}\left(1 - \frac{x_o}{d}\right)\right]\right) \quad \text{for } b_c = 0. \tag{19}$$

In Fig. 5, the allowed and forbidden bands of TE modes for $n_c > n_s$ at $a_c = 10$ are given. It shows that for greater δ and x_o, the TE_0 modes exist only with smaller V_c. Comparing Fig. 5 to Fig. 4 shows that the forbidden bands in the case $n_c > n_s$ are smaller than the bands in the case $n_c > n_s$. Fig. 6 shows that the relations between b_c and V_c have different behavior with the factors δ and x_o.

IV. Conclusion

TE-waves supported by slab waveguides with generalized nonlinear substrate have been investigated. Analytical expressions of the dispersion relations and field-dependent relations are obtained. For the cases of "symmetric" and "asymmetric" structures, the dispersion diagrams are given. If $\delta = 2$ (Kerr-like media), the diagram is the same as those given in the literature. The results show that these structures can support modes with lower frequencies than those in the linear cases. The explicit analytical expression and the dispersion diagrams given here can be used for the design of optical devices based on nonlinear waveguide structures.

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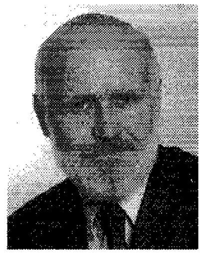

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